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BEHAVIOR OF A COMPETITIVE SYSTEM OF NON-LINEAR RATIONAL DIFFERENCE EQUATIONS

T. F. Ibrahim[1,2]

[1]Department of Mathematics, Applied Faculty (Mahayel) , King Khalid university

[2]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, Egypt.Email: **tfibrahem@mans.edu.eg**

Abstract. The dynamics of a competitive population model specified by a system of nonlinear difference formulas are investigated. We define the system's type of solutions in further detail.

$$I_{n+1} = \frac{I_{n-5}}{\varepsilon + \delta I_{n-5}\ J_{n-2}}, \quad J_{n+1} = \frac{J_{n-5}}{\rho + \sigma J_{n-5}\ I_{n-2}}$$

where $\varepsilon, \delta, \rho, \sigma \in \{-1, 1\}$ with real initials $I_{-3}, I_{-4}, I_{-5}, I_{-1}, I_0, I_{-2}, J_{-1}, J_{-4}, J_{-2}, J_{-5}, J_{-3}$ and J_0 such that $\varepsilon + \delta I_{k-3} J_k \neq 0$ and $\rho + \sigma J_{k-3} I_k \neq 0$ for all $k = -2, -1, 0, 1, 3, 2$.
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1 Introduction

Difference formulae or different dynamical systems have always been a fascinating research topic. The study of difference formulae poses a significant challenge for mathematicians, but it is also extremely rewarding due to the wide range of applications in both applied and pure mathematics. [24].

The research study of the residential properties of rational difference formulas (see, e.g. [10]–[14]) and difference equation systems (see, e.g., [1], [6], and [9]) have taken into account in recent years.

Consider

$$\rho_{n+1} = h(\rho_n, \eta_n), \quad \eta_{n+1} = k(\rho_n, \eta_n) \tag{1}$$

where $n = 0, 1, 2, \ldots$, $(\rho_0, \eta_0) \in J \subseteq \mathbb{R}^2$, and $(h, k) : J \to J$. This system is said to be *competitive* if $h(\rho, \eta)$ is non-decreasing in ρ and non-increasing in η; and $k(\rho, \eta)$ is non-increasing in ρ and non-decreasing in η.

For other papers, we refer the readers to [2], [3], [4], [5], [7], [8], [15],[16], [19], [20], [21], [25], [26], and [27]-[65].