STABILITY OF THREE SPECIES SYMBIOSIS MODEL WITH DELAY AND STOCHASTIC PERTURBATIONS

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Abstract. In this paper a three-species symbiosis population model with delay and stochastic perturbations is considered. The model is modified by considering more general rate which ensure the existence of at least one nontrivial equilibrium. Conditions for the existence of positive equilibrium of the considered model are obtained. New sufficient conditions of stability in probability for the obtained positive equilibrium are formulated in the terms of linear matrix inequalities (LMIs), which can be investigated by virtue of MATLAB. Besides some necessary stability conditions are formulated in the form of simple analytical inequalities. The results obtained are illustrated via numerical simulations of a solution of the considered model.

Keywords. Stochastic differential equations; Positive equilibrium; Stability in probability; Asymptotic mean square stability; Linear matrix inequalities (LMIs).

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1   Introduction

The term symbiosis was first coined by Heinrich Anton de Bary which is termed as the living together of unlike organisms. The relationship of parasitic, mutualistic or commensal in nature can be considered as symbiotic relationship [2]. Such relationships are core of several mathematical models. The classical Lotka-Volterra model has been extensively used to describe the interaction of species. It is also evident that delay is quite natural in ecological modeling as it describe the maturity period. There are huge literature on the population model with delay, especially models on two species, for more details, we refer to [8, 3, 9, 12, 17, 14] and references therein. There are also many three species models [1, 23, 20] and references therein. Delay naturally arises in the mathematical models due to maturation period. There are several works in this direction, for example, He in [11] considered a delayed