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DISCRETIZATION SCHEME OF FRACTIONAL PARABOLIC EQUATION WITH NONLOCAL COEFFICIENT AND UNKNOWN FLUX ON THE DIRICHLET BOUNDARY

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Abstract. This article presents a fractional parabolic equation with an unknown boundary condition and a nonlocal coefficient. Using Rothe method combined with finite elements method and an additional integral measurement, we will reconstruct the missing Dirichlet condition. Numerically, the nonlocal term poses difficulties because the obtained Jacobian matrix is complete. In order to remedy these difficulties we develop an idea given by Gudi in [11]. Finally, a numerical experiments are given demonstrate the effectiveness of the proposed approach.

Keywords. Fractional diffusion equation , Unknown Dirichlet condition, Nonlocal term.

AMS (MOS) subject classification: 35D30, 35R11, 65M20, 65M22.

1 Introduction

Let Ω be an open bounded set of $\mathbb{R}^n$ with Lipschitz continuous boundary Γ. Fixing a final time $T > 0$, we set $I = [0, T]$ and $\Gamma = \overline{\Gamma_D} \cup \overline{\Gamma_N}$, $\Gamma_D \cap \Gamma_N =$ $(meas\,(\Gamma_D) > 0)$, where Γ_D and Γ_N are tow open subsets of Γ and ν is the outward normal vector at each point of Γ. Denote by a a function from $\mathbb{R}$ into $\mathbb{R}$ such that a is continuous. Our aim is to investigate the following inverse problem of identifying the missing Direchlet condition $\gamma(t)$.

$$\begin{cases} D^{\alpha}_{RL} u\,(t,x) - a(l(u)) \triangle u\,(t,x) = f\,(t,x) & in\ I \times \Omega \\ u\,(0,x) = u_0\,(x)\ \ in\ \Omega \\ \nabla u.\nu = g,\ on\ I \times \Gamma_N \\ u = \gamma\,(t)\,,\ on\ I \times \Gamma_D, \end{cases} \tag{1}$$

from an additional measurement of type

$$\int_{\Omega} I^{1-\alpha}\,(u\,(t,x))\,dx = \theta\,(t)\,, \tag{2}$$