ON SOLVABILITY OF A THREE-DIMENSIONAL SYSTEM OF NONLINEAR DIFFERENCE EQUATIONS

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Abstract. In the present study, we consider the three-dimensional system of difference equations
$$x_{n+1} = \frac{a + y_n z_n}{y_n + z_n}, y_{n+1} = \frac{a + z_n x_n}{z_n + x_n}, z_{n+1} = \frac{a + x_n y_n}{x_n + y_n},$$
where $n = 0, 1, \ldots$, $a \in [0, \infty)$ and the initial values x_0, y_0, z_0 are arbitrary real numbers. We show that the mentioned system is solvable. Additionally we describe the set of initial values which produce well-defined solutions of the system and determine behavior of the well-defined solutions.

Keywords. Asymptotic behavior; well-defined solution; forbidden set; nonlinear difference equation; system of difference equations.

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1 Introduction

Nonlinear difference equations are an important class of difference equations. Studying of this kind of equations have recently attracted great interest. One can see this in some recent studies. See, for example [6, 7, 11, 21, 27, 28, 30, 36, 37, 38]. Particularly, there have been a renewed interest on solvable ones of such equations and systems. For example, published papers on solvability of some nonlinear difference equations and some systems can be found in the references [9, 10, 39, 44].

The following nonlinear difference equation

$$x_{n+1} = \frac{a + x_n x_{n-1}}{x_n + x_{n-1}}, \ n \in \mathbb{N}_0, \tag{1}$$

where $a \in [0, \infty)$ and the initial values x_{-1}, x_0 are real numbers, was posed by Kocić and Ladas in [19]. Later, this equation was solved by Rhouma in [23]. See the reference [24] for a more comprehensive study of Eq. (1). In [25] and [26], Stević and Tollu showed that the following systems of difference equations

$$x_{n+1} = \frac{a + p_n q_{n-1}}{p_n + q_{n-1}}, \ y_{n+1} = \frac{a + r_n s_{n-1}}{r_n + s_{n-1}}, \ n \in \mathbb{N}_0,$$